

Reconstructing the interaction rate in holographic models of dark energy

Anjan A. Sen^{*1} and Diego Pavón^{†2}

¹*Center for Theoretical Physics, Jamia Millia Islamia, New Delhi 110025, India*

²*Department of Physics, Autonomous University of Barcelona, 08193 Bellaterra (Barcelona), Spain*

We reconstruct the interaction rate of the holographic dark energy model recently proposed by Zimdahl and Pavón [1] in the redshift interval $0 < z < 1.8$ with observational data from supernovae type Ia, baryon acoustic oscillations, gas mass fraction in galaxy clusters, and the growth factor. It shows a reasonable behavior in the sense that it increases with expansion from a small or vanishing value in the long past but starts decreasing at recent times. The later feature suggests that the equation of state parameter of dark energy does not cross the phantom divide.

I. INTRODUCTION

Nowadays it is widely agreed that our Universe is currently accelerating its expansion -see [2] for recent reviews. However, this consensus does not extend to the agent behind this acceleration and in fact there are many competing candidates. The simplest one, a tiny cosmological constant, does nicely well at the pragmatic observational level- but it entails seemingly unsurmountable problems on the theoretical side. This is why many researchers are considering other possibilities, particularly some or other scalar of tachyon field with a strong negative pressure high enough to drive cosmic acceleration -these go under the collective name of “dark energy” fields.

Among the most recent generic proposals there is a very suggestive one based on the holographic principle. Loosely speaking, the latter asserts that the entropy of a system is given by the number of degrees of freedom lying on the surface that bounds it, rather than in its volume [3]. The roots of this principle are to be found in the thermodynamics of black holes [4]. Nevertheless, as noted by Cohen *et al.* [5], a system may satisfy the holographic principle and, however, include states for which its Schwarzschild radius is larger than system size, L . This can be avoided by imposing the constraint that the energy of the system should not exceed that of black hole of the same size or, equivalently, $\rho \leq 3c^2/(8\pi G L^2)$, where c^2 is a (non-necessarily constant) parameter. In the cosmological context L is usually taken either as the event horizon radius or the Hubble radius. (For a quick summary of holographic dark energy see section 3 of Ref. [1]).

The model of Ref. [1] is based in two main assumptions, (i) dark energy complies with the holographic principle with L identified as the radius of the Hubble horizon, H^{-1} , hence $\rho_x = 3c^2 H^2/(8\pi G)$, and (ii) dark energy and dark matter do not evolve separately but they interact. Accordingly, the energy conservation equations are

$$\dot{\rho}_m + 3H\rho_m = Q, \quad \dot{\rho}_x + 3H(1+w)\rho_x = -Q, \quad (1)$$

where w is the equation of state parameter of dark energy, p_x/ρ_x , which is not constrained to be a constant. Subscripts m and x are for dark matter and dark energy, respectively.

It should be noted that for spatially flat universes in the absence of interaction, $Q = 0$, there would be no acceleration [1, 6]. Besides, Q must be a positive-definite quantity for the coincidence problem [7] to be solved (or at least alleviated) [8], and the second law of thermodynamics to be fulfilled [9]. Further, it has been forcefully argued that the Layzer-Irvine equation [10] when applied to galaxy clusters reveals the existence of the interaction [11]. To the best of our knowledge, the interaction hypothesis was first introduced, well ahead

^{*} E-mail address: anjan.ctp@jmi.ac.in

[†] E-mail address: diego.pavon@uab.es

of the discovery of late acceleration, by Wetterich [12] to reduce the theoretical huge value of the cosmological constant, and was first used in connection to holography by Horvat [13]. As we write, the body of literature on the subject is steadily growing -see [1] and references therein. Most cosmological models implicitly assume that matter and dark energy couple gravitationally only. However, unless there exists an underlying symmetry that would set Q to zero (such a symmetry is still to be discovered) there is no a priori reason to discard the interaction. Ultimately, observation will tell us whether the interaction exists.

Following [1], we will write the interaction as $Q = \rho_x \Gamma$, where Γ is an unknown, semi-positive definite, function that measures the rate at which energy is transferred from dark energy to dark matter. Clearly, as long as the nature of both dark ingredients of the cosmic substratum remain unknown, Γ cannot be derived from first principles; however, one can resort to observational data (in our case, supernovae type Ia (SN Ia), baryon acoustic oscillations (BAO), gas mass fraction in galaxy clusters and the growth factor) to roughly reconstruct it. The next section focuses on reconstructing the dimensionless quantity $\Gamma/3H$.

II. RECONSTRUCTION

The evolution equation

$$\dot{r} = (1+r) \left[3Hw \frac{r}{1+r} + \Gamma \right] \quad (2)$$

for the ratio $r \equiv \rho_m/\rho_x$ between the energy densities follows from Eqs. (1) and the above expressions for ρ_x and Q . With the help of Friedmann equation $\Omega_m + \Omega_x + \Omega_k = 1$, in terms of the usual density parameters $\Omega_i = 8\pi G\rho_i/(3H^2)$ ($i = m, x$), and $\Omega_k = -k/(a^2 H^2)$, where k stands for the spatial curvature index of the Friedmann-Robertson-Walker metric, we can write

$$\dot{r} = -2H \frac{\Omega_k}{\Omega_x} q \quad (3)$$

with $q = -\ddot{a}/(aH^2)$, the deceleration parameter. Here we have assumed $\rho_x \propto H^2$ for holographic dark energy assuming the horizon as the Hubble horizon.

Likewise, starting from the first of Eqs. (1) and using Friedmann equation, we get for the equation of state parameter the expression

$$w(z) = (1+r) \left[\frac{2}{3} \frac{H'}{H} - 1 \right] - \frac{2}{3} \frac{\Omega_k}{\Omega_x} \left[1 - (1+z) \frac{H'}{H} \right], \quad (4)$$

where z denotes the redshift factor and a prime indicates derivative with respect to this quantity.

We fit the Chevallier-Polarsky-Linder parametrization [14], namely,

$$w(z) = w_0 + w_1 \frac{z}{1+z}, \quad (5)$$

where w_0 is the present value of $w(z)$, and w_1 a further constant, to current data from different observational probes and subsequently use the fitting values for w_0 and w_1 to reconstruct the dimensionless ratio $\Gamma/3H$.

As for the data, we resort to the various SN Ia observations in recent times. In particular we use 60 Essence supernovae [15], 57 SNLS (Supernova Legacy Survey) and 45 nearby supernovae. We have also included the new data release of 30 SNe Ia detected by the Hubble Space Telescope and classified as the Gold sample by

Riess *et al.* [15]. The combined data set can be found in Ref. [16]. The total number of data points involved is 192.

Next we add the measurement of the CMB (Cosmic Microwave Background) acoustic scale at $z_{BAO} = 0.35$ as observed by the SDSS (Sloan Digital Sky Survey) for the large scale structure. This is the Baryon Acoustic Oscillation (BAO) peak) [17].

We also consider the gas mass fraction of galaxy cluster, $f_{gas} = M_{gas}/M_{tot}$, inferred from the X-ray observations [18]. This depends on the angular diameter distance d_A to the cluster as $f_{gas} = d_A^{3/2}$. The number of data point involved is 26.

Likewise, the 2dF galaxy redshift survey has measured the two point correlation function at an effective redshift of $z_s = 0.15$. This correlation function is affected by systematic differences between redshift space and real space measurements due to the peculiar velocities of galaxies. Such distortions are expressed through the redshift distortion parameter β . Correlation function can be used to measure it as $\beta = 0.49 \pm 0.09$ at the effective redshift of $z = 0.15$ of the 2dF survey. This result can be combined with linear bias parameter $b = 1.04 \pm 0.11$ obtained from the skewness induced in the bispectrum of the 2dFGRS by linear biasing to find the growth factor g at $z = 0.15$, namely $g = 0.51 \pm 0.11$ [19].

A. The spatially flat case

The simplest case is when $\Omega_k = 0$. Then, from (3), $r = r_0$, where the zero subscript means present value, and equations (2) and (4) reduce to

$$\frac{\Gamma}{3H} = -r_0 \left[\frac{2}{3}(1+z)\frac{H'}{H} - 1 \right], \quad (6)$$

and

$$w(z) = (1 + r_0) \left[\frac{2}{3}(1+z)\frac{H'}{H} - 1 \right], \quad (7)$$

respectively. Using these two expressions we determine w_0 and w_1 from the data and, with them, we reconstruct $\Gamma/3H$ -see figures 1 and 2.

The best fit values, with 1σ error bars for the parameters when all the data (SN Ia + BAO + x-rays + growth factor) are included, come to be: $w_0 = -1.13 \pm 0.24$, $w_1 = 0.66 \pm 1.35$ (for $\Omega_{m0} = 0.25$ & $\Omega_k = 0$, Fig. 1); and $w_0 = -0.80 \pm 0.28$, $w_1 = -1.75 \pm 1.79$ (for $\Omega_{m0} = 0.3$ & $\Omega_k = 0$, Fig. 2).

Here, one cautionary remark seems in order. The fact that r was never large might lead the reader to think that the model of Ref. [1] seriously conflicts with the standard scenario of cosmic structure formation. One may believe that at early times the amount of dark matter would have been too short to produce gravitational potential wells deep enough to lead to the condensation of galaxies. However, this is not so; a matter dominated phase is naturally recovered since at high and moderate redshifts the interaction is even smaller than at present whence the equation of state of the dark energy becomes close to that of non-relativistic matter -see [1] for details.

A related point is to realize that dark energy clusters similarly to dark matter when the equation of state of the former stays close to that of the latter. In this connection, it is worthwhile to recall the perturbation dynamics of this model. This was studied in [1] making use of the perturbed metric

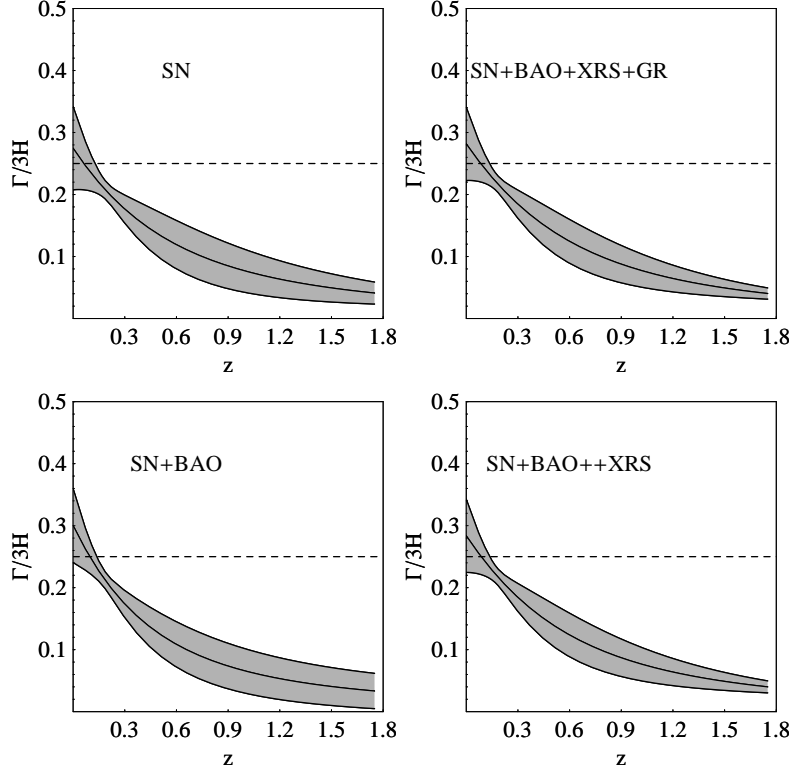


FIG. 1: The dimensionless ratio $\Gamma/(3H)$ vs redshift. In the four panels we have fixed $\Omega_{m0} = 0.25$ and $\Omega_k = 0$. The solid line is for the mean value and the shaded area indicates the 1σ region. The region above the horizontal dashed line can be visited only when the dark energy becomes of phantom type, i.e., $w < -1$.

$ds^2 = -(1 + 2\psi) dt^2 + a^2 (1 - 2\psi) \delta_{\alpha\beta} dx^\alpha dx^\beta$, with ψ the scalar metric perturbation, and the Bardeen gauge-invariant variable [20]

$$\zeta \equiv -\psi + \frac{1}{3} \frac{\hat{\rho}}{\rho + p} = -\psi - H \frac{\hat{\rho}}{\dot{\rho}}, \quad (8)$$

which represents curvature perturbations on hypersurfaces of constant energy density. Here, an upper-hat means perturbation of the corresponding quantity; likewise, $\rho = \rho_m + \rho_x$ and $p = p_x$.

Corresponding quantities for the components are

$$\zeta_A \equiv -\psi - H \frac{\hat{\rho}_A}{\dot{\rho}_A} \quad (A = m, x). \quad (9)$$

On large perturbation scales we have that

$$\dot{\zeta} = -H \left(P - \frac{\dot{p}}{\dot{\rho}} D \right) \quad (10)$$

with $P \equiv \hat{p}/(\rho + p)$ and $D \equiv \hat{\rho}/(\rho + p)$ and parallel expressions for ζ_m and ζ_x . Therefore, insofar as both equations of state do not differ significantly the evolution of these two perturbations will be alike.

In the particular, simplest, case of $\Gamma = \text{constant}$ last equation reduces to

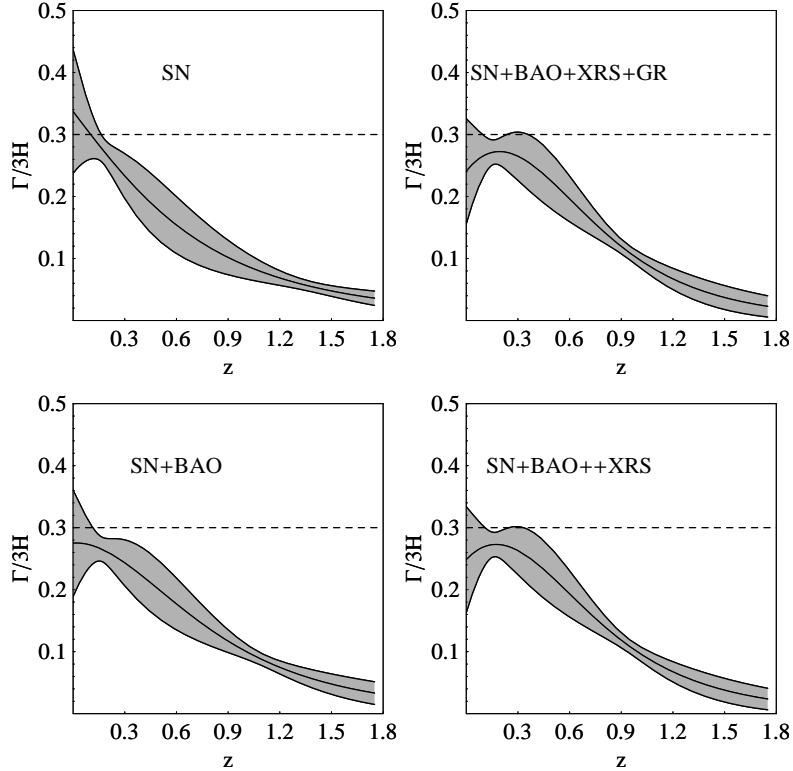


FIG. 2: Same as Fig. 1 except that here we have fixed $\Omega_{m0} = 0.30$ and $\Omega_k = 0$.

$$\dot{\zeta} = -\frac{\Gamma}{6(1+\frac{p}{\rho})} \frac{\hat{r}}{r^2}. \quad (11)$$

This one can be readily integrated to (see section 6 of Ref. [1] for details)

$$\zeta = \zeta_i - \frac{\Gamma \hat{r}}{3 r 3 H_i r - \Gamma} \frac{1}{\left[\left(\frac{a}{a_i} \right)^{3/2} - 1 \right]}. \quad (12)$$

Again, so long as the the equation of state parameter of dark energy w remains close to that of dark matter, both components will cluster in a similar fashion. Further, a non-vanishing interaction introduces a non-adiabatic feature that grows as $a^{3/2}$ which will have an impact on the integrated Sachs-Wolfe effect. Possibly this feature might be used in the future to discriminate the model under consideration from the Λ CDM model -recall that in the latter ζ remains constant and does not produce the said effect.

B. Non-spatially flat cases

When $\Omega_k \neq 0$ the ratio r between energy densities is no longer a constant. This is an extra unknown function in our fitting procedure. But one should not expect a large variation in r in the redshift range that has been considered in this paper. In our subsequent computation, we take the Taylor series expansion for r around its present day value and take up to the first order term in the expansion. We therefore parameterize it as

$$r = r_0 + r_1(1 - a) = r_0 + r_1 \frac{z}{1+z}, \quad (13)$$

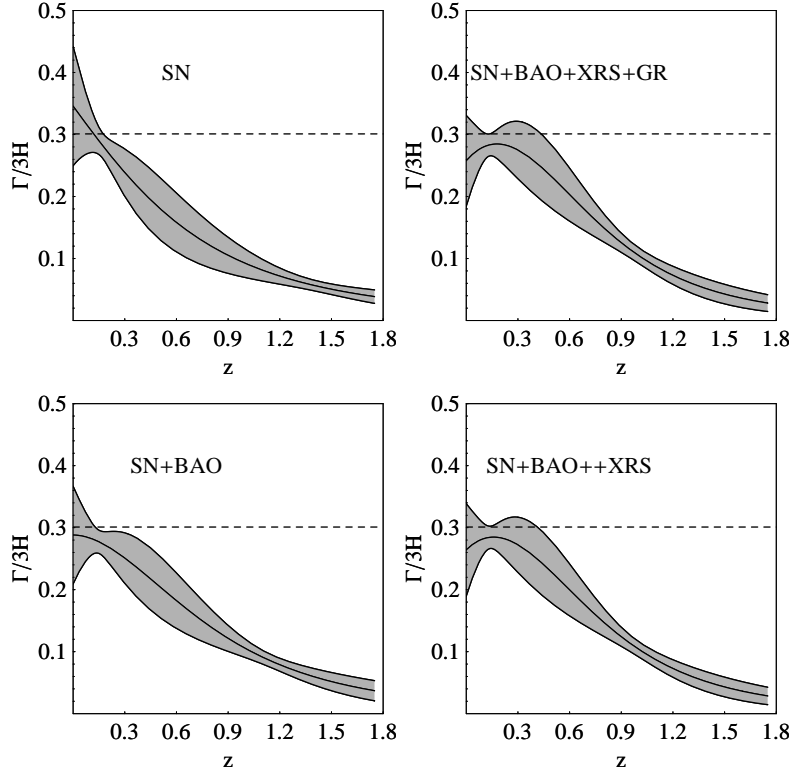


FIG. 3: Same as Fig. 1 except that we have fixed $\Omega_{m0} = 0.30$ and $\Omega_{k0} = 0.002$.

with r_0 is the present day value for r . r_1 is a constant which can be related to the present ratio of densities between Ω_k and Ω_x by

$$\frac{\Omega_{k0}}{\Omega_{x0}} = -\frac{r_1}{2} \left[1 - \left(\frac{H'}{H} \right)_{z=0} \right]. \quad (14)$$

This can be used to fix the unknown constant r_1 for a given Ω_{k0} and Ω_{x0} . Also

$$\frac{\Gamma}{3H} = -\frac{1}{1+r} \left(r' \frac{1+z}{3} - w r \right) \quad (15)$$

where w is given by Eq. (4).

Using these expressions, the ratio $\Gamma/3H$ is reconstructed from the data in Fig. 3.

The best fit values, with 1σ error bars for the parameters when all the data (SN Ia + BAO + x-rays + growth factor) are included, come to be: $w_0 = -0.806 \pm 0.29$, $w_1 = -1.74 \pm 3.33$. It is seen that the curvature, being small as WMAP 3yr [21] tells us, has little consequence on the evolution of the interaction rate.

III. CONCLUDING REMARKS

We reconstructed the interaction term Q of Ref. [1] in the redshift interval ($0 < z < 1.8$) of observational data (supernovae type Ia, baryon acoustic oscillations, gas mass fraction, and growth factor). The interaction rate Γ (and hence Q) is always positive, its general trend is to decrease as z increases but it shows no indication of becoming negative at larger redshifts. This corroborates that as previously suggested [9, 11] the energy transfer proceeds from dark energy to dark matter rather than otherwise. While phantom behavior cannot be excluded at recent and present times it only occurs in a manifest way either for large Ω_{x0} -see Fig. 1- or when just the supernovae data are considered (top-left panel of Figs. 1-3). When Ω_{x0} is a bit lower (say, 0.7) and BAO and other data are included, the mean value of dimensionless interaction rate, $\Gamma/3H$, no longer crosses the phantom divide (i.e., the horizontal dashed line). It simply reaches a maximum near $z = 0$ and decreases with expansion. This is a reassuring result as holography is not compatible with phantom energy [22]. On the other hand, it should be noted that Ω_{x0} values as high as 0.75 do not seem favored from a combination of results from WMAP 1yr and weak lensing which yields $\Omega_{x0} = 0.70 \pm 0.3$ [23].

Adding a small curvature term -say, $\Omega_{k0} = 0.002$ -, has only a tiny impact (compare Figs. 2 and 3). This also holds true when the curvature bears the opposite sign; this is why we have not included a corresponding figure.

In any case, it should be noted that the concordance Λ CDM model ($w_0 = -1$, $w_1 = 0$) shows compatibility within 1σ confidence level with the set of data considered in this work.

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